QUASI-STATIONARY THERMAL REGIME ARISING DURING PERIODIC PULSED HEATING

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We theoretically and experimentally study the quasi-stationary thermal regime which arises during periodic pulsed heating by laser radiation.

The thermophysical features of periodic-pulsed heating have been intensively studied in connection with the invention of lasers which operate in the periodic-pulsed regime [1-3]. Their promise in measuring the thermophysical properties of materials was pointed out in [4]. However, this goal requires knowledge of the exact solution of the variable components in conditions of the quasi-stationary thermal regime, that is, for large times t. The description of periodic-pulsed heating in [5] neglected heat loss α from radiation at the surface of the sample. The solution of the problem in the interval $0 \leq t \leq t_d$ was reduced to the solution of that for a continuous heat source of intensity q_0 and was constructed over the interval $t_d \leq t \leq t_p$ with the use of a fictive sink of the same intensity, shifted with respect to the continuous source by an amount t_d. Such an approach was used in [6] in an investigation of the initial stages of pulsed-periodic heating when the number of pulses is still comparatively low. It has been shown [6-8] that the smooth part of the solution (averaged temperature) coincides with the solution of the problem of heating by continuous sources with intensity $\bar{q} = \gamma q_0$, where $\gamma = t_d/t_p$ is the duty factor, that is, the stationary part of the solution is absent.

In order to separate the stationary and oscillatory parts of the solution in the conditions established in the quasi-stationary regime, it is necessary to account for heat loss α from the surface of the sample [4, 9]. Moreover, the temperature pulsations which arise during the action of periodic pulses, unlike those arising from single pulses, propagate with a finite speed determined by the propagation speed of the temperature wave with the frequency of the fundamental harmonic $\omega = 2\pi/t_p$. For this reason, the method of computing the pulsations [5] at large times may not be completely reliable. With this in mind, the goal of this work is to study the quasi-stationary state of the temperature field $T(t, x) = \bar{T}(x) + u(t, x)$, $|u|/\bar{T} \ll 1$, which arises during periodic pulsed heating by surface heat sources, with linearized heat losses by radiation from the sample surface:

$$\left(-\lambda \frac{\partial u}{\partial x} + \alpha u\right)\Big|_{x=0} = -\bar{q} + q(t),$$

where $\alpha = 4\sigma\varepsilon \bar{T}^3(0)$; $\bar{q} = \sigma\varepsilon[\bar{T}^4(0) - T_0^4] - \lambda \bar{T}'_x(0)$.

It has been shown that the function $u_1(t) = u(t, 0)$ admits the representation $u_1(t) = \theta_1(t) + v_1(t)$, where $\theta_1(t)$ is the oscillatory regime with the same frequency spectrum as that of the heat source $q(t)$. The $v_1(t)$ is residual terms tending to zero for $t \to \infty$. Since the linearization procedure is approximate, an experimental verification of the adequacy of the proposed mathematical model for a realistic heating process was conducted. This model was developed for the case of periodic pulsed Gaussian heat source.

To obtain the computational formulae, we study the oscillatory regime in a linear model:

$$\frac{\partial u}{\partial t} = a\frac{\partial^2 u}{\partial x^2};\ u = u(t, x);\ 0 \leqslant x < \infty;\ 0 \leqslant t < \infty,$$
$$u(0, x) = 0;\ u(t, \infty) = 0, \tag{1}$$
$$\left(-\lambda\frac{\partial u}{\partial x} + \alpha u\right)\Big|_{x=0} = \tilde{q}(t);\ \tilde{q}(t) = -\bar{q} + q(t);\ q(t) = AI(t).$$

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The function q(t) is assumed to be piecewise smooth with period t_p. We find a solution for an established quasi-stationary thermal regime, that is, an asymptote of the function $u_1(t) = u(t, 0)$ for $t \to \infty$. We denote by $\bar{c}_k$ and c_k the Fourier coefficients of the functions $\bar{q}(t)$ and q(t) in terms of the system $\{\exp(i2\pi kt/t_p)\}$ on the interval $0 \le t \le t_p$. Clearly

$$\bar{c}_0 = -\bar{q} + c_0; \quad \bar{c}_k = c_k = t_p^{-1} \int_0^{t_p} q(\tau) \exp(i\omega_k \tau)\, d\tau \qquad (2)$$

$$(\omega_k = 2\pi k/t_p;\ k \neq 1).$$

Also, let $\tilde{q}(p)$ be the transform of the original $\eta(t)\cdot q(t)$, where $\eta(t)$ is the Heaviside step function. By using the operational calculus, we obtain a representation of $u_1(t)$ from (1) with the help of the contour integral:

$$u_1(t) = \frac{D^{-1/2}}{2\pi i} \int_{p_0 - i\infty}^{p_0 + i\infty} \frac{\exp(pt)}{\sqrt{p} + \sqrt{\mu}} \tilde{q}(p)\, dp \quad (p_0 > 0;\ \mu = \alpha^2/D),$$

$$\sqrt{p} = \sqrt{|p|} \exp\left\{\frac{i}{2} \arg p\right\};\ p \in G: |\arg p| < \pi. \qquad (3)$$

According to the formula for transforming the periodic original and using (2), we obtain

$$\tilde{q}(p) = F(p)[1 - \exp(-pt_p)]^{-1} = \frac{\bar{c}_0}{p} + \sum_{k \neq 0} \frac{c_k}{p - i\omega_k},$$

$$F(p) = \int_0^{t_p} \bar{q}(\tau) \exp(-p\tau)\, d\tau. \qquad (4)$$

From (3) and (4) we find:

$$u_1(t) = \theta_1(t) + \bar{c}_0/\alpha + v_1(t),$$

$$\theta_1(t) = D^{-1/2} \sum_{k \neq 0} \operatorname*{Res}_{p - i\omega_k} \frac{\exp(pt)}{\sqrt{p} + \sqrt{\mu}} \tilde{q}(p), \qquad (5)$$

$$v_1(t) = \int_0^{\infty} \frac{f(\sigma)\, d\sigma}{\sqrt{\sigma} \exp(\sigma t)}; \quad f(\sigma) = \frac{D^{-1/2} \sigma F(-\sigma)}{\pi(\mu + \sigma)[1 - \exp(\sigma t_p)]}.$$

It is evident that at $\sigma = 0$ the function $f(\sigma)$ has a removable singularity. From this and directly from the expression for $v_1(t)$, we conclude that $v_1(t) = O(1)$ $(t \to \infty)$. Further, by computing the residue, we have

$$u_1(t) = \theta_1(t) + \bar{c}_0/\alpha + O(1), \qquad (6)$$

$$\theta_1(t) = \frac{1}{\alpha} \sum_{k \neq 0} \frac{c_k \exp(i\omega_k t)}{1 + h\sqrt{ik}} = \frac{2}{\alpha} \sum_{k=1}^{\infty} \operatorname{Re} \frac{c_k \exp(i\omega_k t)}{1 + h\sqrt{ik}}. \qquad (7)$$

Here $h = \alpha^{-1} \sqrt{\omega D}$; $\omega = 2\pi/t_p$

Now, the condition that the limiting quasi-stationary regime described by u(t, x) exists leads to

$$\bar{c}_0 = 0; \quad \bar{q} = t_p^{-1} \int_0^{t_p} q(\tau)\, d\tau.$$

We introduce an expression for the oscillatory regime $\theta_1(t)$ in the case when the heat flux density q(t) has the form:

$$q(t) = \begin{cases} q_0, & 0 \le t \le t_d, \quad q(t + \nu t_p) = q(t); \\ 0, & t_d < t < t_p, \quad \nu = 1, 2, \ldots \end{cases}$$

Using this definition we find from (2) the Fourier coefficients:

$$\bar{c}_0 = -\bar{q} + \gamma q_0 \quad (\gamma = t_d/t_p),$$
$$\bar{c}_k = c_k = q_0 \frac{\sin\psi_k}{\pi k} \exp(-i\psi_k) \quad (k \neq 0, \ \psi_k = \pi k \gamma).$$

Substituting the values $\bar{c}_k = c_k$ for $k \neq 0$ into (7), after transforming we obtain

$$\theta_1(t) = \frac{2\theta_0}{\pi} \sum_{k=1}^{\infty} A_k(h) \sin(\omega_k t + \varphi_k), \quad \theta_0 = \frac{q_0}{\alpha}; \tag{8}$$

$A_k(h) = k^{-1}(1 + h\sqrt{2k} + kh^2)^{-1/2} \sin\psi_k$; $\varphi_k = \operatorname{arctg}(1 + h^{-1}(k/2)^{-1/2}) - \psi_k$. Carrying out similar operations for $x \neq 0$, we have

$$\theta(t, x) = \frac{2\theta_0}{\pi} \sum_{k=1}^{\infty} A_k(h) \times \exp(-x\sqrt{\omega_k/2a}) \sin(\omega_k t + \varphi_k - x\sqrt{\omega_k/2a}). \tag{9}$$

Formula (9) represents the solution to the problem of heating by a periodic pulsed heat source in the form of a linear superposition of temperature waves of frequency $\omega_k = 2\pi/t_p$. The propagation speed of the temperature pulsation is bounded and does not exceed the propagation speed of the fundamental-frequency temperature wave: $v = \sqrt{2a\omega}$. With increasing distance x from the sample surface, the high-frequency harmonics are suppressed and the heating becomes nearly sinusoidal. Figure 1 illustrates the difference in the described forms of temperature pulsations with the approach used in [5] (broken curve) and that developed here (solid curve). The pulsation form $\theta_1(t)$ at the surface of a tungsten sample are shown, computed using (8) and the formula given in [5, 10] for three values of γ: 0.2, 0.5, and 0.8. The mean temperature was taken as $\bar{T} = 1600$ K, and the pulse repetition period as 100 μsec. We used data from [11] for values of the thermophysical constants. It is clear that for $\gamma \lesssim 0.2$, both approaches give similar results for the heating stage. In this case the temperature change coincides with that computed for the action of a single pulse

$$\theta_1(t) = 2q_0\sqrt{t/\pi D} \quad (0 \leqslant t \leqslant t_d). \tag{10}$$

In the heating stage, the discrepancy between the solid and dashed curves increases with increasing γ. On the other hand, during the cooling stage $t_d < t < t_p$, the difference between the values of $\theta_1(t)$ decreases with increasing γ. In the case considered here, heat loss does not exceed 1%, the parameter $h \gg 1$ and when computing the form of the temperature pulsation we can set

$$A_k(h) \simeq \sin\psi_k / k\sqrt{\omega_k D}; \quad \varphi_k \simeq \frac{\pi}{4} - \psi_k, \tag{11}$$

that is, the form of the pulsation for both computational methods is virtually independent of temperature. It follows that the discrepancy in the curves in Fig. 1 is due to differences in the approach to computing $\theta(t, x)$.

In connection with this, the question arises of the criteria for the transition of periodic heating (curves 2, 3) to periodic pulses whose temperature in the heating stage coinciees with that calculated by (10) (curve 1). For this, using (8) and (11) we write an expression for the amplitude of the k-th harmonic:

$$\theta_k \simeq 2q_0 \sin\psi_k / \pi k\sqrt{\omega_k D}. \tag{12}$$

This makes obvious the role of the thermophysical property (the coefficient of thermal activity) during periodic pulsed heating: through it the connection between pulses and waves characterizing the heating process is established. Indeed, by equating the values of D in (10) and (12) we obtain

$$\theta_k (k\omega_k^{1/2} \sin^{-1}\psi_k) = \theta_a (\pi t_d)^{-1/2}, \tag{13}$$

which for $\gamma \ll 1$ takes the form

$$\theta_k \simeq (\theta_a \sqrt{\gamma/2}) k^{-1/2}. \tag{14}$$

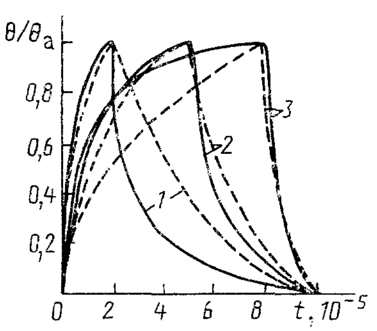

Fig. 1

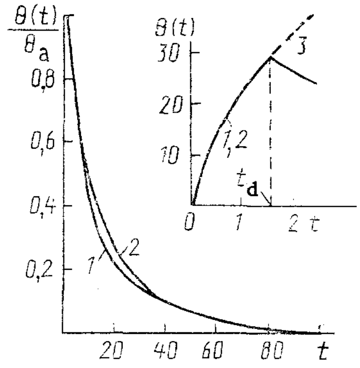

Fig. 2

Fig. 1. Dependence of the normalized form of the temperature pulsation on the duty factor γ for two computational methods: solid curves correspond to the representation of $\theta(t)$ as a superposition of temperature oscillations; the dashed curves were constructed from the solution for pulsed heat sources. Curves 1, 2, 3 are for γ = 0.2, 0.5, 0.8, respectively. t is in sec.

Fig. 2. Form of the temperature pulsation which arises during periodic pulsed heating by laser radiation. $\bar{T}_m$ = 1600 K; t_d = 1.6 μsec; t_p = 100 μsec; material is tungsten. The heating stage is shown in the upper corner. (1) theoretical curve; (2) experimental; dashed curve (3) corresponds to single pulse heating. θ is in K, t is in μsec.

Expressions (13) and (14) can be considered as criteria for the establishment of the periodic pulsed heating regime. Here $\theta_a = \theta_1(t_d)$ is the amplitude of the temperature pulsations.

To verify the suitability of the mathematical model (7) for realistic heating processes, we performed an experimental study of the quasi-stationary thermal regime which arises when a tungsten sample is heated by radiation from an IAG laser LTI-502. The experimental procedure did not differ from that described in [9]. The duration and repetition frequency of the laser-generated pulses was 1.6 μsec and 10 kHz, the average radiated power 8.6 W, and the diameter of the heating spot was 500 μm. This work regime heated the sample to a mean temperature $\bar{T}_m$ = 1600 K and produced a pulsation amplitude of $\theta_a \simeq 29.6$ K. From Fig. 2 it is evident that during the heating stage, the form of the pulsation obtained experimentally (curve 2) coincides with that calculated from (10) (curve 3). The difference between the theoretical (curve 1) and experimental relations during the cooling stage is connected with the approximations used when linearizing the original problem.

Figure 3 shows the values of θ_k obtained by expanding the experimental dependence $\theta_1(t)$ in a Fourier series in terms of the frequencies $\omega_k = 2\pi k/t_p$. Since $\gamma = 1.6\cdot10^{-2} \ll 1$, the dependence of the amplitude of the harmonic on its number must be recitifed in θ_k, $k^{-\frac{1}{2}}$ coordinates, with the tangent of the angle of inclination proportional to the pulsation amplitude. It is clear that (14) is indeed fulfilled, and the graphically determined pulsation amplitude of $\theta_a \simeq 31.1$ K with ~5% error coincides with that measured experimentally. Thus, for periodic pulsed heating, the form of the pulsation may be completely reconstructed in terms of the characteristic amplitude.

We will touch on the question of the kinetics of the approach to the quasistationary thermal regime and the duration of the transition process. As an example, Fig. 4 shows the time dependence of the mean temperature in the heating spot $\bar{T}(t)$ during establishment of the described quasi-stationary thermal regime of $\bar{T}_m$ = 1600 K. Clearly, the approach to the quasi-stationary regime takes place in three stages. In the first stage $0 \leq t \lesssim 0.2t_q$, the mean temperature in the heating spot $\bar{T}(t) - T_0$ grows proportionally to $\sqrt{t}$, and in this case the form of the pulsation is well described within the framework of the linear model [6, 10]. Here $t_q \simeq 6.3\cdot10^2$ sec is the approach time to the quasi-stationary regime. The linearization method considered in this work corresponds to the third heating stage $0.7t_q \lesssim t \leq t_q$. In this stage, the difference between the stationary and mean temperature in the heating spot $\bar{T}_m - \bar{T}(t)$ decreases as $t^{-3/2}$, and $\theta_1(t)$ is described by (8). In the second stage of the heating

$0.2t_q \lesssim t \lesssim 0.7t_q$, heat loss is nonnegligible and cannot be linearized, so to find the pulsation form, one must know the solution to the thermal conductivity equation with nonlinear boundary conditions.

We will show that the proposed method allows us to study the quasi-stationary thermal regime in the general case, when the distribution of the absorbed power density along the surface z = 0 has the form [12]

$$Q(t, x, y) = q(t)\exp\left(-\frac{x^2+y^2}{2r^2}\right)$$

Initial stage of heating was examined in [8] in connection with an estimate of the influence of the bunched pulse structure of lasers operating in the free generation regime on the mean heating spot temperature $\bar{T}(t)$. For simplicity, we limit the discussion to square pulse forms. We write the system modeling the thermal process in dimensionless form

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial\bar{x}^2} + \frac{\partial^2\theta}{\partial\bar{y}^2} + \frac{\partial^2\theta}{\partial\bar{z}^2}; \ \theta = \theta(\tau, \bar{x}, \bar{y}, \bar{z}),$$

$$\left(\theta - \frac{\partial\theta}{\partial\bar{z}}\right)_{\bar{z}=0} = \bar{q}(\tau)\exp\left(-\frac{\bar{x}^2+\bar{y}^2}{2\sigma^2}\right),$$

$$\tau \geqslant 0; \ -\infty < \bar{x} < \infty; \ -\infty < \bar{y} < \infty; \ \bar{z} \geqslant 0,$$

where

$$\bar{x} = x/l_0; \ \bar{y} = y/l_0; \ \bar{z} = z/l_0; \ \sigma = r/l_0; \ \tau = \omega_0 t; \ \omega_0 = a/\alpha^2; \ l_0 = \lambda/\alpha.$$

Let

$$\theta(p, \xi, \eta, \bar{z}) = \int_0^\infty d\tau \iint_{-\infty}^{\infty} \theta(\tau, \bar{x}, \bar{y}, \bar{z})\exp(-p\tau - i\xi\bar{x} - i\eta\bar{y})\, d\bar{x}d\bar{y}.$$

Then

$$\theta = c(\mu)\exp(-\mu\bar{z}); \ \mu = (\xi^2+\eta^2+p)^{1/2}.$$

The coefficient $c(\mu)$ is found from the boundary condition

$$c(\mu) = \frac{2\pi\sigma^2\bar{\bar{q}}(p)}{1+\sqrt{\xi^2+\eta^2+p}}\exp\left\{-\frac{\sigma^2}{2}(\xi^2+\eta^2)\right\}.$$

Using the inverse formulas, we write the solution in the form

$$\theta(\tau, \bar{x}, \bar{y}, \bar{z}) = \frac{1}{2\pi i}\int_{p_0-i\infty}^{p_0+i\infty} dp\left\{\bar{\bar{q}}(p)\exp(p\tau) \times \frac{1}{4\pi^2}\iint_{-\infty}^{\infty} f(\sqrt{\xi^2+\eta^2})\exp(i\xi\bar{x}+i\eta\bar{y})\,d\xi d\eta\right\} (p_0 > 0).$$

Here

$$f(\sqrt{\xi^2+\eta^2}) = \frac{2\pi\sigma^2}{1+\sqrt{\xi^2+\eta^2+p}}\exp\left\{-\frac{\sigma^2}{2}(\xi^2+\eta^2) - \bar{z}\sqrt{\xi^2+\eta^2+p}\right\}$$

and by the transform formula of the periodic original

$$\bar{\bar{q}}(p) = \frac{1}{p}\,\frac{q_1-(q_1+q_2)\exp(-\gamma p t_p)+q_2\exp(-pt_p)}{1-\exp(-pt_p)},$$

$$q_0 = q_1+q_2; \ q_2 = q_1\gamma/(1-\gamma).$$

Transforming the inner integral and computing the residue, we finally obtain the asymptote to the solution for $t \to \infty$

$$\theta(\tau, \bar{x}, \bar{y}, \bar{z}) = \frac{1}{2\pi}\int_0^\infty \rho I_0(\rho\sqrt{\bar{x}^2+\bar{y}^2})\, A(\rho, \bar{z})\, d\rho, \tag{15}$$

where

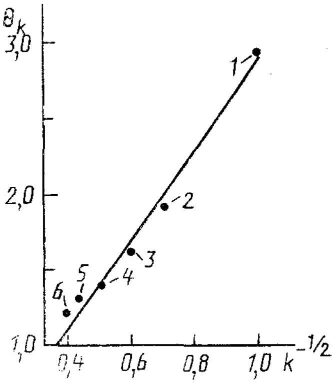

Fig. 3

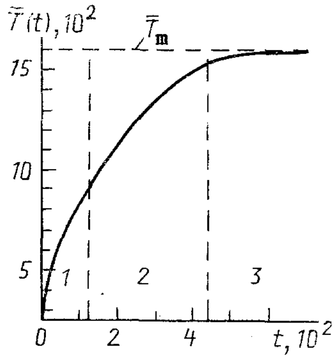

Fig. 4

Fig. 3. Rectification of the dependence of the amplitude of harmonic θ_k on number k in θ_k, $k^{-1/2}$ coordinates: the numbers 1-6 correspond to the harmonic number.

Fig. 4. Kinetics of the approach of the mean temperature in the heating spot $\bar{T}(t)$ to the stationary value $\bar{T}_m$ = 1600 K: area 1) initial heating stage; 2) transition stage; 3) stage of establishment of quasi-stationary thermal regime. The initial temperature is T_0 = 297 K; t_d = 1.6 μsec; t_p = 100 μsec; material is tungsten. $\bar{T}$ is in K.

$$A(\rho, \bar{z}) = \frac{2\theta_0}{\pi} \sum_{k=1}^{\infty} \frac{\sin\psi_k}{k} \operatorname{Re}\left\{\frac{\exp(-\bar{z}\sqrt{i\bar{\omega}_k + \rho^2})}{1 + \sqrt{i\bar{\omega}_k + \rho^2}} \exp\left[i(\omega_k t - \psi_k)\right]\right\};$$

$I_0(\rho\sqrt{\bar{x}^2 + \bar{y}^2})$ is the zeroth order Bessel function of the first type; $\bar{\omega}_k = \omega_k/\omega_0$; $\rho = \sqrt{\xi^2 + \eta^2}$.

Analysis of (15) by computer shows that as in the one-dimensional case, the coincidence of the pulsation form with that computed from the pulsed representation [8] is observed only for $\gamma \lesssim \gamma_{cr}$ in the heating stage. The value of γ_{cr} for which coincidence is still observed lies in the range $0.05 \lesssim \gamma_{cr} \lesssim 0.2$. Here the upper bound corresponds to $\sigma \to \infty$, where the isotherms are flat, the lower bound to $\sigma \approx 0$, where isotherms have spherical symmetry.

Thus we have worked out a method for computing the quasi-stationary thermal regime which arises during periodic pulsed heating, based on a representation of the temperature pulsations as a superposition of temperature waves. We have shown that there is satisfactory agreement between the proposed mathematical model and a realistic heating process.

NOTATION

T, temperature; $\bar{T}$ and θ, mean value and the oscillatory component of the temperature; a and λ, thermal diffusivity and theraml conductivity coefficients; $D = \lambda c_p \rho$, square of the thermal activity; α, coefficient of heat loss; A, the absorbance; I and q, intensity and absorbing power density of the laser radiation; q_0, heat flux density in a pulse; t_d and t_p, length and repetition period of the laser generation pulses.

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USE OF SENSITIVITY FUNCTIONS IN THE PROBLEM OF DESIGNING A MULTILAYER HEAT SHIELD

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An approach is proposed for solving the problem of designing a multilayer heat shield with a prescribed structure from the restrictions on its temperature.

The solution of a problem of the form

$$M=\sum_{j=1}^{n}\rho_{\text{var},j}h_{\text{var},j}\to\min_{\bar{h}_{\text{var}}},\tag{1}$$

$$T_{\text{con},i}\leqslant\hat{T}_{\text{con},i},\ i=\overline{1,m},\tag{2}$$

$$h_{\text{var},j}\geqslant\check{h}_{\text{var},j}\tag{3}$$

gives the weighted-optimal solution of the problem of designing a one-dimensional multilayered construction (packet) of a prescribed structure, which is exposed to a high-temperature medium and is characterized by restrictions on the temperature in separate zones of the structure. The temperature in the packet is described by the one-dimensional Fourier equation [1]

$$\rho c(T)\frac{\partial T}{\partial\tau}=\frac{\partial}{\partial y}\left(\lambda(T)\frac{\partial T}{\partial y}\right).\tag{4}$$

With the help of the method of the penalty functions [2] the starting problem (1)-(3) can be reduced to an unconditional-minimization problem

$$F=\sum_{j=1}^{n}\rho_{\text{var},j}h_{\text{var},j}+\sum_{i=1}^{m}a_i\max(0,\ T_{\text{con},i}-\hat{T}_{\text{con},i})+\sum_{j=1}^{n}b_j\max(0,\ \check{h}_{\text{var},j}-h_{\text{var},j})\to\min_{\bar{h}_{\text{var}}}.\tag{5}$$

The difficulties arising in the development of methods for solving problems of this kind are discussed in [3, 4]. However, these methods are not widely employed for investigating practical design questions. A simplified approach to the synthesis of structures, based on finding the combination of thicknesses of m separate layers such that conditions of the type

$$\varphi_i(h_{\text{var},1},\ \ldots,\ h_{\text{var},m})=T_{\text{con},i}(h_{\text{var},1},\ \ldots,\ h_{\text{var},m})-\hat{T}_{\text{con},i}=0,\ i=\overline{1,m}\tag{6}$$

are satisfied, is employed much more often.

The present paper is devoted to methodological questions concerning the construction of the solution to problems of the type formulated above.

One possible algorithm for solving the problem (6) by iteration consists of the following sequence of operations which are performed at each k-th iteration:

formation of the initial approximation $h^{(k)}_{\text{var},j}$ $(j=\overline{1,m})$ for the unknown thicknesses of the layers;

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